5

Control System Design in State-Space

5.1 Design: Classical vs. Modern

A fashion designer tailors the apparel to meet the tastes of fashionable people, keeping in mind the desired fitting, season and the occasion for which the clothes are to be worn. Similarly, a control system engineer *designs* a control system to meet the desired objectives, keeping in mind issues such as where and how the control system is to be implemented. We need a control system because we do not like the way a plant behaves, and by *designing* a control system we try to *modify* the behavior of the plant to suit our needs. *Design* refers to the process of changing a control system's *parameters* to meet the specified stability, performance, and robustness objectives. The design *parameters* can be the unknown constants in a *controller's* transfer function, or its state-space representation. In Chapter 1 we compared open- and closed-loop control systems, and saw how a closed-loop control system has a better chance of achieving the desired performance. In Chapter 2 we saw how the classical transfer function approach can be used to design a closed-loop control system, i.e. the use of graphical methods such as Bode, Nyquist, and root-locus plots. Generally, the classical design consists of varying the controller transfer function until a desired closed-loop performance is achieved. The classical indicators of the closed-loop performance are the closed-loop frequency response, or the locations of the closed-loop poles. For a large order system, by varying a limited number of constants in the controller transfer function, we can vary in a pre-specified manner the locations of only a few of the closed-loop poles, *but not all of them*. This is a major limitation of the classical design approach. The following example illustrates some of the limitations of the classical design method.

Example 5.1

Let us try to design a closed-loop control system for the following plant transfer function in order to achieve a zero steady-state error when the desired output is the unit step function, $u_s(t)$:

$$G(s) = (s + 1)/[(s - 1)(s + 2)(s + 3)] \tag{5.1}$$

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The single-input, single-output plant, $G(s)$, has poles located at $s = 1$, $s = -2$, and $s = -3$. Clearly, the plant is unstable due to a pole, $s = 1$, in the right-half s-plane. Also, the plant is of type 0. For achieving a zero steady-state error, we need to do *two things*: make the closed-loop system stable, and make type of the closed-loop system at least unity. Selecting a closed-loop arrangement of Figure 2.32, both of these requirements are apparently met by the following choice of the controller transfer function, $H(s)$:

$$H(s) = K(s-1)/s \tag{5.2}$$

Such a controller would apparently cancel the plant's unstable pole at $s = 1$ by a zero at the same location in the closed-loop transfer function, and make the system of type 1 by having a pole at $s = 0$ in the open-loop transfer function. The open-loop transfer function, $G(s)H(s)$, is then the following:

$$G(s)H(s) = K(s+1)/[s(s+2)(s+3)] \tag{5.3}$$

and the closed-loop transfer function is given by

$$\begin{aligned} Y(s)/Y_d(s) &= G(s)H(s)/[1 + G(s)H(s)] \\ &= K(s+1)/[s(s+2)(s+3) + K(s+1)] \end{aligned} \tag{5.4}$$

From Eq. (5.4), it is apparent that the closed-loop system can be made stable by selecting those value of the design parameter, K, such that all the closed-loop poles lie in the left-half s-plane. The root-locus of the closed-loop system is plotted in Figure 5.1 as K

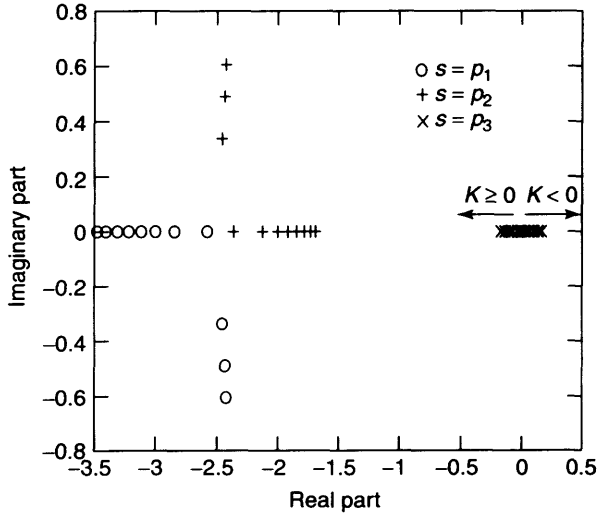

Figure 5.1 Apparent loci of the closed-loop poles of Example 5.1 as the classical design parameter, K, is varied from −1 to 1

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is varied from -1 to 1. Apparently, from Figure 5.1, the closed-loop system is stable for $K \geq 0$ and unstable for $K < 0$. In Figure 5.1, it appears that the pole $s = p_3$ determines the stability of the closed-loop system, since it is the pole which crosses into the right-half s-plane for $K < 0$. This pole is called the *dominant pole* of the system, because being closest to the imaginary axis it *dominates* the system's response (recall from Chapter 4 that the smaller the real part magnitude of a pole, the longer its contribution to the system's response persists). By choosing an appropriate value of K, we can place only the dominant pole, $s = p_3$, at a desired location in the left-half s-plane. The locations of the other two poles, $s = p_1$ and $s = p_2$, would then be governed by such a choice of K. In other words, by choosing the sole parameter K, the locations of all the three poles cannot be chosen *independently of each other*. Since all the poles contribute to the closed-loop performance, the classical design approach may fail to achieve the desired performance objectives when only a few poles are being directly affected in the design process.

Furthermore, the chosen design approach of Example 5.1 is misleading, because it *fails to even stabilize* the closed-loop system! Note that the closed-loop transfer function given by Eq. (5.4) is of *third order*, whereas we expect that it should be of *fourth order*, because the closed-loop system is obtained by combining a third order plant with a first order controller. This discrepancy in the closed-loop transfer function's order has happened due to our attempt to *cancel* a pole with a zero at the same location. Such an attempt is, however, doomed to fail as shown by a state-space analysis of the closed-loop system.

Example 5.2

Let us find a state-space representation of the closed-loop system designed using the classical approach in Example 5.1. Since the closed-loop system is of the configuration shown in Figure 3.7(c), we can readily obtain its state-space representation using the methods of Chapter 3. The Jordan canonical form of the plant, $G(s)$, is given by the following state coefficient matrices:

$$\mathbf{A_p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}; \quad \mathbf{B_p} = \begin{bmatrix} 0 \\ 1/3 \\ -1/2 \end{bmatrix}$$

$$\mathbf{C_p} = [1 \quad 1 \quad 1]; \quad \mathbf{D_p} = 0 \tag{5.5}$$

A state-space representation of the controller, $H(s)$, is the following:

$$\mathbf{A_c} = 0; \quad \mathbf{B_c} = K; \quad \mathbf{C_c} = -1; \quad \mathbf{D_c} = K \tag{5.6}$$

Therefore, on substituting Eqs. (5.5) and (5.6) into Eqs. (3.146)–(3.148), we get the following state-space representation of the closed-loop system:

$$\mathbf{x}^{(1)}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{y_d}(t) \tag{5.7}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{y_d}(t) \tag{5.8}$$

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