# 8

# Digital Control Systems

## 8.1 What are Digital Systems?

So far, we have confined our attention to *continuous time* (or *analog*) systems, i.e. systems whose inputs and outputs are *continuous* functions of time. However, we often have to deal with systems whose output signals are *discontinuous* in time. Such systems are called *discrete time* (or *digital*) systems. Where are digital systems encountered? Any continuous time signal that is processed (or *sampled*) by either a human being, or by a machine – mechanical or electrical – becomes a digital signal by the very process of *sampling*, because a human being or a machine takes some *non-zero* time, albeit small, to *evaluate* or *record* the value of the signal at any given time. Over this non-zero time interval, the current value of the signal is *held* and analyzed, before the value of the signal at the *next* time interval is processed. Hence, by sampling a continuous time signal, we essentially *convert* it into a digital signal, and the *sampling process* is a digital system. Another type of digital system is the one which produces discontinuous outputs due to the intrinsic nature of its mechanism. Watches and clocks of any type, tick and tock in a *discontinuous* manner (i.e. the needle or electronic display moves in *jerks* rather than in a perfectly smooth manner) while trying to simulate a *continuous* passage of time. The jerks or discontinuities in the motion of a time-piece can be minimized by choosing an appropriate mechanism. However, the *time interval* between two successive ticks, no matter how small, is non-zero for an atomic clock as well as for a grandfather clock. Hence, we have to deal with inherently digital systems, as well as digital systems obtained by sampling continuous time signals. However, if the *sampling interval* of the output of a continuous system, or the discontinuity in time of the output of an inherently digital system, is *small* in comparison to the *time constants* (i.e. inverse of natural frequencies) of the system, then such a signal, even though digital by nature, can be *approximated* to be a continuous signal, and we can apply all the methods developed in the previous chapters to study and design a control system that produces such an output. Interestingly, the *solution* of the state-equations for an analog system requires its *approximation* by a corresponding *digital* system, as we saw in Chapter 4!

Most mechanical and electrical control systems before the advent of the digital computer had previously been approximated as continuous time (or *analog*) systems. Nowadays, control systems are invariably based on the *digital computer*, which is an *inherently* digital system, because it receives and produces *electrical impulses*, rather than continuous electrical signals, on which *binary arithmetic* is based. There are several advantages in having a digital computer as a part of the control system (usually the feedback controller), namely the ease and versatility of implementing complex control laws merely by programming the computer, relative insensitivity to noise, and low cost. Hence, modern control systems employ digital computers in almost all applications, such as automobiles, pacemakers for heart, aircraft and spacecraft, electronic appliances, personal computers, robotic manipulators, chemical and nuclear reactors and industrial processes. Such applications are usually approximated as an analog plant controlled by a digital computer. However, by having a digital computer in the control system for controlling an analog plant, we introduce an additional characteristic to the control system – the *sampling rate*, i.e. the *inverse* of the non-zero time interval required by the digital computer to process the analog output at each instant of time. The *sampling rate* can drastically modify the behavior of a control system – such as stability, performance, and robustness – when compared to an analog control system. In this chapter, we study how to take into account the sampling rate of digital control systems while studying stability, performance, and robustness, and how such control systems can be designed.

A system into which a continuous time signal is input and from which a digital signal is output (perhaps due to processing by a digital computer) is called an *analog-to-digital converter* (or *A/D converter*). A system which converts digital inputs to analog outputs is called a *digital-to-analog* (or D/A) converter. Since a digital computer processes only digital input signals and produces digital outputs, while using a digital computer in a feedback control system for controlling an analog plant, we must have *both* A/D and D/A converters, as shown in Figure 8.1. However, whereas D/A conversion is a simple and continuous time process (where the analog output is a weighted sum of a set of input impulses), the A/D conversion is a digital process consisting of *sampling* and *holding* the analog input (as pointed out above), and producing an impulse as the output.

We discussed in Chapter 4 how a good approximation of an analog system by a digital system can be obtained if the discrete time step used to solve the digitized state-equations is appropriately small. Conversely, the *sampling rate* at which an analog signal is sampled is crucial for the successful implementation of a digital control system. The *sampling rate* must not be smaller than the *bandwidth* (i.e. the range of frequencies in the power spectrum) of the analog signal, otherwise a *distorted* digital output will be produced. Thus, we may regard the A/D converter as a *low-pass filter* (see Example 7.2) with sampling rate as the cut-off frequency. To reduce signal distortion, the cut-off frequency (sampling rate) should be appreciably larger than the bandwidth of the signal we wish to pass through the filter. Generally, the sampling rate should be at least *twice the bandwidth*

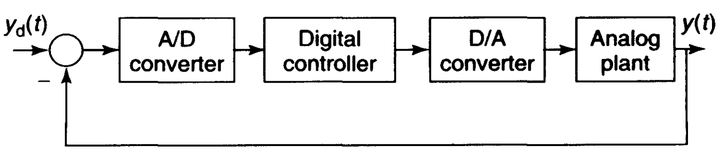


**Figure 8.1** A block diagram of a negative feedback control system with an analog plant and a digital controller