Answers to Selected Exercises

Chapter 2

2.4. The system is stable with $\omega_n = 377.27$ rad/s and $\zeta = 1.26 \times 10^{-6}$. Maximum overshoot = 0.0069 m, steady state deviation = 0.007 m, settling time = 6100 seconds.

2.5. The system is stable with $\omega_n = 5634.7$ rad/s and $\zeta = 1.409 \times 10^{-6}$. Maximum overshoot = 3.13×10^{-5} m, steady state deviation = 3.15×10^{-5} m, settling time = 460 seconds.

2.6. (a) $G(i\omega) = 1/(-m\omega^2 + ic\omega + k)$.

(b) $\omega_n = (k/m)^{1/2}$; $\zeta = c/[2(km)^{1/2}]$.

(c) $s_{1,2} = \pm i(k/m)^{1/2}$.

2.8. (a) Maximum overshoot = 0; settling time = 0.8 second; steady state output = 0.2.

(b) Maximum overshoot = −80%; settling time = 5.33 seconds; steady state output = 0.5.

(c) Maximum overshoot = 464%; settling time = 18.5 seconds; steady state output = 1.0.

2.9. Maximum overshoot = 213.3 units; steady state output = 0.

2.10. $|\delta_{\max}| = 250$ m.

2.14. (a) $K_I = 1$, $K_D = 2.114$, $K_P = 2.394$;

(b) maximum overshoot = 21.6%; settling time = 6.33 seconds; steady state error = 0;

(c) $0 \le K_I \le 5.8$;

(d) Gain margin = ∞, phase margin = 129.15°, gain-crossover frequency = 1.9 rad/s.

2.18. (a) $G(s) = K(s + a)/[s^2(s + 1)]$, $H(s) = K_1 + K_2 s + K_3 a s(s + 1)/(s + a)$.

(b) Type = 2.

(c) Plant is unstable.

(d) $H(s) = [K_3 a s^2 + s(K_1 + K_2 + K_3 a) + K_1 a]/(s + a)$.

(e) For $K_1 = 500$, $\omega_n = 1.83$ rad/s, $\zeta = 0.753$.

(f) Maximum overshoot = 0; settling time = 1.5×10^{-3} second; steady state error = 0.

(g) Gain margin = ∞; phase margin = 178.04°; gain crossover frequency = 102.47 rad/s.

2.22. (a) Gain margin = ∞; phase margin = 3.63°; gain crossover frequency = 31.67 rad/s.

(b) Phase is maximum at $\omega = 400$ rad/s; change in gain = 40 dB.

(c) Plant: max. overshoot = 21%, settling time = 3.8 sec., steady state error = −199.
Closed-loop system: max. overshoot = 32.5%, settling time = 0.25 sec., steady state error = 0.005.

(d) Gain margin = ∞, phase margin = 99.27°, gain crossover frequency = 43.88 rad/s.

2.25. (a) Gain margin = 28.09 dB, phase margin = 12.64°.

(b) $\omega_o = 0.0775$ rad/s, $\alpha = 0.1$.

(c) Plant: maximum overshoot = 91%, settling time = 2000 seconds.
Closed-loop system: maximum overshoot = 42%, settling time = 70 seconds.

2.28. $K = 0.7$, maximum overshoot = 1.8 m, settling time = 12.25 seconds. $|z(t)|_{\max} = 29.9$ m/s^2.

Plant: gain margin = ∞, phase margin = 20.98°, gain crossover freq. = 377.3 rad/s.
Closed-loop: gain margin = ∞, phase margin = ∞.

Chapter 3

3.1. (a) $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2/17 & -10/17 & 0 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/17 \end{bmatrix}$; $\mathbf{C} = [0 \quad 2 \quad 5]$; $\mathbf{D} = 0$.

3.2. (a) $\mathbf{A} = \begin{bmatrix} -4 & -3 & 1 & -5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$; $\mathbf{C} = [0 \quad 1 \quad -3 \quad 1]$; $\mathbf{D} = 0$.

3.4. $x_1^{(1)}(t) = x_2(t)$; $x_3(t)x_2^{(1)}(t) = -2x_2(t)x_3^{(1)}(t) + a_{\mathrm{Mc}}(t)$; $\theta_M(t) = x_1(t)$.

3.8. (a) $\mathbf{A} = \begin{bmatrix} -0.5 & 1.0724 & 0 & 0 \\ -1.0724 & -0.5 & 0 & 0 \\ 0 & 0 & -0.0025 & 0.0774 \\ 0 & 0 & -0.0774 & -0.0025 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} -1.6771 \\ -0.6065 \\ 6.8075 \\ 6.3307 \end{bmatrix}$

$\mathbf{C} = \begin{bmatrix} 0.0122 & -0.0073 & 0.0132 & -0.0116 \\ 0.2922 & -0.7779 & -0.0001 & -0.0001 \\ -0.0074 & -0.6646 & -0.0353 & -0.0276 \end{bmatrix}$; $\mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Chapter 4

4.1. (a) $$e^{\mathbf{A}t} = \begin{bmatrix} e^{-2t} & (3e^{-2t} - 2.5607e^{-0.5858t} - 0.4393e^{-3.4142t}) & 0.75(2e^{-2t} - e^{-0.5858t} - e^{-3.4142t}) \\ 0 & (1.2071e^{-0.5858t} - 0.2071e^{-3.4142t}) & 0.3535(e^{-0.5858t} - e^{-3.4142t}) \\ 0 & 0.707(-e^{-0.5858t} + e^{-3.4142t}) & (-0.2071e^{-0.5858t} + 1.2071e^{-3.4142t}) \end{bmatrix}$$

(b) $\lambda_1 = -2$, $\lambda_2 = -0.5858$, $\lambda_3 = -3.4142$. Stable.

(c) $$\mathbf{X}(t) = \begin{bmatrix} 1.45e^{-2t} - 0.1189e^{-0.5858t} - 0.3311e^{-3.4142t} \\ 0.056e^{-0.5858t} - 0.156e^{-3.4142t} \\ -0.0329e^{-0.5858t} + 0.5329e^{-3.4142t} \end{bmatrix}$$

Chapter 5

5.1. (a) Controllable, (b) Uncontrollable.

5.4. (a) Yes. (b) No. No.

5.6. (a) Yes. (b) No. (c) No.

5.8. $$\mathbf{K} = \begin{bmatrix} -0.3814 & -0.1654 & 0.3212 & 0.0615 & -0.6328 & -0.4675 \\ 2.1142 & 0.3183 & -0.3035 & -0.0720 & 1.0523 & 0.5360 \end{bmatrix}$$
Maximum overshoots: $y_1(t)$ (13.6789 m/s^2); $y_2(t)$ (−0.1312 rad/s).
Settling time: 4 seconds.

5.9. $$\mathbf{K} = \begin{bmatrix} -0.0033 & 0.0008 & 0.0003 & -0.0011 \\ 0.5702 & -0.6445 & 9.0813 & 0.2692 \end{bmatrix}$$

5.12. (a) Unobservable. (b) Observable. (c) Unobservable.

5.15. (a) Observable. (b) Unobservable. (c) Unobservable.

5.16. (a) Unobservable. (b) Observable. (c) Observable. (d) Unobservable.

(e) $\mathbf{L} = [\,-15.28; \quad -30.393; \quad 6.9472; \quad 5.4\,]^T$.

5.18. $$\mathbf{L}^{\mathrm{T}} = \begin{bmatrix} 0.0000 & -1.0685 & 4.2603 & -1.9726 \\ 0.0000 & -2.8493 & -1.9726 & -0.2603 \end{bmatrix}$$

Chapter 6

6.2. $$\mathbf{Q} = \begin{bmatrix} 10^{-4} & 0 & 0 & 0 \\ 0 & 10^{-4} & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}; \quad \mathbf{R} = \mathbf{I}$$

$$\mathbf{K} = \begin{bmatrix} 0.0072 & 0.0796 & -0.0254 & -0.0967 \\ 0.0000 & -0.0073 & 0.0967 & -0.0254 \end{bmatrix}$$

6.3. Settling time = 0.1 second. Maximum overshoot = −0.0117 rad/s.
Largest magnitude of $\delta_{\mathrm{A}}(t) = 0.3$ rad. Largest magnitude of $\delta_{\mathrm{R}}(t) = 0.03$ rad.

6.4. $\mathbf{K} = \begin{bmatrix} -0.0077 & -0.0280 & -0.0916 & 0.0269 & -0.0344 & 0.0261 \\ -0.0037 & -0.0597 & 0.1593 & 0.6495 & -1.0641 & 1.0249 \end{bmatrix}$
Settling time: 0.33 second. Maximum overshoots: -9×10^{-4} units ($y_1(t)$); -0.09 units ($y_2(t)$).
Largest magnitude of $u_1(t) = 7.73 \times 10^{-4}$ units. Largest magnitude of $u_2(t) = 5.27 \times 10^{-4}$ units.

6.5. $\mathbf{Q} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$; $\mathbf{R} = \mathbf{I}$

$\mathbf{K} = \begin{bmatrix} 0.3665 & -4.9893 & -4.1433 & 5.7678 & -5.2177 & -3.0179 \\ 3.5047 & 19.3521 & -12.8400 & 11.7067 & 14.2192 & 39.5569 \end{bmatrix}$

6.8. $\mathbf{K}_{o1} = \begin{bmatrix} 0.5983 & 0.0656 & -0.3026 & 0.9993 \\ 0.0626 & -0.7147 & -0.3068 & 0.0363 \end{bmatrix}$; $\mathbf{K}_d = \mathbf{0}$.

6.11. $\mathbf{K}_{o1} = \begin{bmatrix} -3.1623 & -15.1463 & -2.8724 & 7.7314 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.1623 & -15.0165 & -3.1197 & 7.5203 \end{bmatrix}$;
$\mathbf{K}_d = \mathbf{0}$.

6.12. $\mathbf{Q} = 10^{-5}\,\mathbf{I}$, $\mathbf{R} = \mathbf{I}$, $\mathbf{V} = 1000\mathbf{I}$. Maximum magnitude of $u_1(t)$: 0.92 units.
Maximum magnitude of $u_2(t)$: 0.24 units.

6.14. Maximum overshoot of $\theta^{(1)}(t)$: -0.03 rad/s. Final value of $\theta^{(1)}(t) = -0.003$ rad/s.

Chapter 7

7.1. $x_m = 0.5164$, $R_x(0) = 0.0843$, $x_{ms} = 0.3502$.

7.2. $\omega_o = 7$ rad/s.

7.8. $s_1 = -16.0070$, $s_2 = -9.8148$, $s_3 = -2.5413$, $s_4 = -0.9606$, $s_{5,6} = -0.9336 \pm 6.3617i$.

7.12. $\mathbf{F} = \mathbf{B}$, $\mathbf{Z} = \mathbf{C}\mathbf{C}^{\mathrm{T}}$, $\Psi = \mathbf{0}$, $\mathbf{V} = 10^6\mathbf{B}^{\mathrm{T}}\mathbf{B}$.
$\mathbf{L}^{\mathrm{T}} = \begin{bmatrix} 4234.9 & -13\,360 & 10\,714 & -6993.3 & -15\,013 & 20\,143 \\ 5370.1 & -16\,994 & 13\,631 & -8889.8 & -19\,117 & 25\,652 \end{bmatrix}$
Largest magnitude of $u_1(t) = 0.053$ units.
Largest magnitude of $u_2(t) = 1.877$ units.

Chapter 8

8.1. (a) $F(z) = ze^{aT}/(ze^{aT} - 1)$.

8.2. (a) $f(kT) = -0.5025i(0.995i)^k + 0.5025i(-0.995i)^k$.

8.3. (a) $G(z) = z(z - 0.6375)/[(z - 1)(z - 0.8187)]$.

8.4. (a) $G(z) = [z(e^{-T} + 2T - 1) + 1 - e^{-T} - 2Te^{-T}]/[z^2 + 2z(T - 1) + 1 + 2Te^{-T}]$.

8.6. (a) Unstable for all $T > 0$.

8.9. (a) $y(\infty) = \infty$.

8.12. The closed-loop system is stable for $-0.06 < K < 0$. With $K = -0.002$, the closed-loop step response settles in about 1 second, with a maximum overshoot of about 6 per cent.

8.19. $\mathbf{A_d} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$; $\mathbf{B_d} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\mathbf{C_d} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{D_d} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

8.25. $\mathbf{K} = \begin{bmatrix} -0.0904 & 0.0226 & -0.2418 & 0.1948 \\ 0.0491 & 0.4488 & -2.5087 & 0.0211 \end{bmatrix}$

8.30. $\mathbf{K} = \begin{bmatrix} 0.9833 & 0.0899 & 0.0248 & 0.0067 & -0.0800 & -0.0403 \\ 0.1610 & -0.0469 & 0.0109 & -0.0120 & 0.1442 & 0.0762 \end{bmatrix}$

Chapter 9

9.2. (a) $\mu(\mathbf{G}) = 10.24$, (b) $\mu(\mathbf{G}) = 4.65$, (c) $\mu(\mathbf{G}) = 102.8$, (d) $\mu(\mathbf{G}) = 10.8$.

9.3. Maximum value of $\mu(\mathbf{P_{22}}(i\omega))$ is approximately 12.5 occurring at about $\omega = 100$ rad/s.

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