will ultimately hit the aircraft, irrespective of the pilot's inputs to the aircraft. To analyze whether the pilot can escape the missile by maneuvering the aircraft with the help of rudder and aileron inputs, let us find the eigenvalues of the entire system as follows:

```
>> [a,b,c,d]=ssdata(syst); damp(a) <enter>

Eigenvalue                  Damping      Freq. (rad/s)
 3.07e-001+1.02e+000i       -2.89e-001   1.06e+000
 3.07e-001-1.02e+000i       -2.89e-001   1.06e+000
 -7.23e-004                 1.00e+000    7.23e-004
 -8.23e-004                 1.00e+000    8.23e-004
 -1.00e-002                 1.00e+000    1.00e-002
 -1.55e-002                 1.00e+000    1.55e-002
 -1.59e-002                 1.00e+000    1.59e-002
 -2.74e-002+1.13e+000i      2.42e-002    1.13e+000
 -2.74e-002-1.13e+000i      2.42e-002    1.13e+000
 -1.39e-001+9.89e-001i      1.39e-001    9.99e-001
 -1.39e-001-9.89e-001i      1.39e-001    9.99e-001
 -1.50e-001+9.89e-001i      1.50e-001    1.00e+000
 -1.50e-001-9.89e-001i      1.50e-001    1.00e+000
 -3.16e-001                 1.00e+000    3.16e-001
 -3.82e-001                 1.00e+000    3.82e-001
 -5.04e-001                 1.00e+000    5.04e-001
 -5.38e-001                 1.00e+000    5.38e-001
 -9.30e-001                 1.00e+000    9.30e-001
 -1.09e+000                 1.00e+000    1.09e+000
```

The complex conjugate eigenvalues $0.307 \pm 1.02i$ with a positive real part indicate that the system is unstable. Hence, it is possible for the pilot to ultimately escape the missile. The controller, *sysc*, must be re-designed to enable a hit by making the entire system asymptotically stable.

Exercises

3.1. Derive a state-space representation for each of the systems whose governing differential equations are the following, with outputs and inputs denoted by $y_i(t)$ and $u_i(t)$ (if $i > 1$), respectively:

(a) $17d^3y(t)/dt^3 + 10dy(t)/dt - 2y(t) = 2du(t)/dt + 5u(t)$.

(b) $d^2y_1(t)/dt^2 + 3dy_1(t)/dt - 6y_2(t) = -u_1(t)/7$; $-2d^2y_2(t)/dt^2 + 9y_2(t) - dy_1(t)/dt = 5du_1(t)/dt - u_2(t)$.

(c) $100d^4y(t)/dt^4 - 33d^3y(t)/dt^3 + 12d^2y(t)/dt^2 + 8y(t) = 27du_1(t)/dt - u_1(t) + 5u_2(t)$.

(d) $d^5y_1(t)/dt + 9y_1(t) - 7d^2y_2(t)/dt^2 + dy_2(t)/dt = 2d^3u(t)/dt^3 - 16d^2u(t)/dt^2 + 47du(t)/dt + 3u(t)$.

3.2. Derive a state-space representation for the systems whose transfer matrices are the following:

(a) $Y(s)/U(s) = (s^2 - 3s + 1)/(s^5 + 4s^3 + 3s^2 - s + 5)$.

(b) $\mathbf{Y}(s)/\mathbf{U}(s) = [(s+1)/(s^2+2s+3) \quad s/(s+3) \quad 1/(s^3+5)]$.

(c) $\mathbf{Y}(s)/\mathbf{U}(s) = \begin{bmatrix} 1/(s+2) & 0 \\ -1/(s^2+3s) & (s+4)/(s+7) \end{bmatrix}$

3.3. Derive a state-space representation for a satellite orbiting a planet (Example 2.3). Linearize the nonlinear state-space representation for small deviations from a circular orbit.

3.4. For a missile guided by *beam-rider* guidance law (Eq. (2.19)), derive a state-space representation considering the commanded missile acceleration, $a_{\mathrm{Mc}}(t)$, as the input, and the missile's angular position, $\theta_{\mathrm{M}}(t)$, as the output.

3.5. For the closed-loop *beam-rider* guidance of a missile shown in Figure 2.8, derive a state-space representation if the target's angular position, $\theta_{\mathrm{T}}(t)$, is the input, and the missile's angular position, $\theta_{\mathrm{M}}(t)$, is the output.

3.6. For a missile guided by the *command line-of-sight* guidance law (Eq. (2.20)), derive a state-space representation considering the commanded missile acceleration, $a_{\mathrm{Mc}}(t)$, as the input, and the missile's angular position, $\theta_{\mathrm{M}}(t)$, as the output.

3.7. For the closed-loop *command line-of-sight* guidance of a missile shown in Figure 2.9, derive a state-space representation if the target's angular position, $\theta_{\mathrm{T}}(t)$, is the input, and the missile's angular position, $\theta_{\mathrm{M}}(t)$, is the output. Can the state-space representation be linearized about an equilibrium point?

3.8. Derive a state-space representation for the longitudinal dynamics of an aircraft (Example 2.10) with elevator deflection, $\delta(t)$, as the input, and $[v(t) \quad \alpha(t) \quad \theta(t)]^T$ as the output vector. Convert the state-space representation into:

(a) the Jordan canonical form,

(b) the controller companion form,

(c) the observer companion form.

3.9. Derive a state-space representation for the compensated closed-loop chemical plant of Example 2.25, with the closed-loop transfer function given by Eq. (2.159). Convert the state-space representation into:

(a) the Jordan canonical form,

(b) the controller companion form,

(c) the observer canonical form.