

by a *first-order actuator* with a *time-constant* of 0.05 s, and a *dead-zone* in the interval $-0.05 < t < 0.05$. The actuator is saturated at $\delta(t) = \pm 0.3$ rad. A nonlinear feedback control system is devised using a switch which allows a feedback gain of 1.9 when the roll-rate, $p(t) = \dot{\phi}^{(1)}(t)$, is greater than or equal to 1 rad/s, and a unity feedback otherwise. A nominal cascade compensator gain of 0.1 is used to achieve desirable response (which is different from that designed in Example 2.24 due to the additional actuator dynamics). Figure 4.21 shows the SIMULINK block diagram and the resulting step response of the closed-loop system. Note that a zero overshoot, a zero steady-state error, zero oscillation, and a settling time of less than two seconds have been achieved, despite the nonlinear control system dynamics. A maximum roll-rate of about 3 rad/s is observed. This performance is representative of modern fighter aircraft, where the ability to achieve a large steady bank angle quickly and without any overshoots or oscillations is an important measure of the aircraft's *dog-fighting* maneuverability.

SIMULINK has many advanced features for simulating a complex control system, such as the creation of new sub-system blocks and *masking blocks* through M-files, C programs, or SIMULINK block diagrams, for effortless integration in your system's model. This allows an extension of the SIMULINK graphical functions to suit your own needs of analysis and design. The SIMULINK *demos* and the User's Guide for SIMULINK [7] are very helpful in explaining the advanced usage and extension of SIMULINK block library.

Exercises

- 4.1. A homogeneous linear, time-invariant system is described by the following state-dynamics matrix:

$$\mathbf{A} = \begin{bmatrix} -2 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -4 \end{bmatrix} \quad (4.95)$$

- Find the state-transition matrix of the system.
- Find the eigenvalues of the system. Is the system stable?
- Calculate the response, $\mathbf{x}(t)$, of the system to the initial condition, $\mathbf{x}(0) = [1; -0.1; 0.5]^T$.

- 4.2. For a linear, time-invariant system described by the following state-space representation:

$$\mathbf{A} = \begin{bmatrix} -1.5 & 0.2 \\ 0.13 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \quad (4.96)$$

calculate the response, $\mathbf{x}(t)$, if the initial condition is $\mathbf{x}(0) = [10; 2]^T$, and the input vector is $\mathbf{u}(t) = [t; 1]^T$.

4.3. The lateral dynamics of an aircraft are described by the following state-equation:

$$\begin{bmatrix} p^{(1)}(t) \\ r^{(1)}(t) \\ \beta^{(1)}(t) \\ \phi^{(1)}(t) \end{bmatrix} = \begin{bmatrix} -15 & 0 & -15 & 0 \\ 0 & -0.8 & 10 & 0 \\ 0 & -1 & -0.8 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p(t) \\ r(t) \\ \beta(t) \\ \phi(t) \end{bmatrix} + \begin{bmatrix} 25 & 3 \\ 0 & -3.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_A(t) \\ \delta_R(t) \end{bmatrix} \quad (4.97)$$

- (a) Determine the step response, $\mathbf{x}(t) = [p(t); r(t); \beta(t); \phi(t)]^T$, to the *aileron* input, $\delta_A(t) = u_s(t)$.
- (b) Determine the step response, $\mathbf{x}(t) = [p(t); r(t); \beta(t); \phi(t)]^T$, to the *rudder* input, $\delta_R(t) = u_s(t)$.
- (c) Determine the response, $\mathbf{x}(t) = [p(t); r(t); \beta(t); \phi(t)]^T$, if the initial condition is $p(0) = 0.1$ rad/s, $r(0) = \beta(0) = \phi(0) = 0$, and the two inputs are zeros, $\delta_A(t) = \delta_R(t) = 0$.
- (d) Determine the response, $\mathbf{y}(t) = [p(t); \phi(t)]^T$, if the initial condition is zero, $\mathbf{x}(0) = \mathbf{0}$, and the input vector is $\mathbf{u}(t) = [\delta_A(t); \delta_R(t)]^T = [0.1\delta(t); -0.1\delta(t)]^T$.

4.4. For the compensated closed-loop chemical plant of Example 2.25 (for which a state-space representation was obtained in Exercise 3.9), determine the output, $y(t)$, and the plant input, $u(t)$, if the desired output, $y_d(t)$, is a *unit impulse function*, and the initial condition is zero.

4.5. For the multivariable closed-loop system of Exercise 2.30 (for which a state-space representation was obtained in Exercise 3.13) with controller parameter $K = 1$, determine the output, $\mathbf{y}(t)$, and the plant input, $\mathbf{u}(t)$, if the desired output is $\mathbf{y}_d(t) = [u_s(t); -u_s(t)]^T$, and the initial condition is zero.

4.6 A linear time-varying system has the following state-space representation:

$$\mathbf{A}(t) = \begin{bmatrix} -5/13 & 1 \\ 1 & -5/13 \end{bmatrix}; \mathbf{B}(t) = \begin{bmatrix} e^{t/2} \\ e^{-t} \end{bmatrix} \quad (4.98)$$

Calculate the response, $\mathbf{x}(t)$, if the input is a unit step function and the initial condition is $\mathbf{x}(0) = [-1; 1]^T$.

4.7. For the missile of Example 4.11, compute:

- (a) the step response to $\alpha(t) = u_s(t)$.
- (b) the step response to $\Delta L(t) = u_s(t)$.
- (c) the initial response, $\mathbf{x}(t)$, to the initial condition, $\mathbf{x}(0) = [100; -10; 100; 0; 0; 0]^T$ and zero input, $\mathbf{u}(t) = \mathbf{0}$.

4.8. Solve the *van der Pol* equation (Eq. (2.204)) of Exercise 2.2, if $a = 5$ and $b = 3$, and the initial condition is given by $x^{(1)}(0) = -0.1$, and $x(0) = 0.5$.