

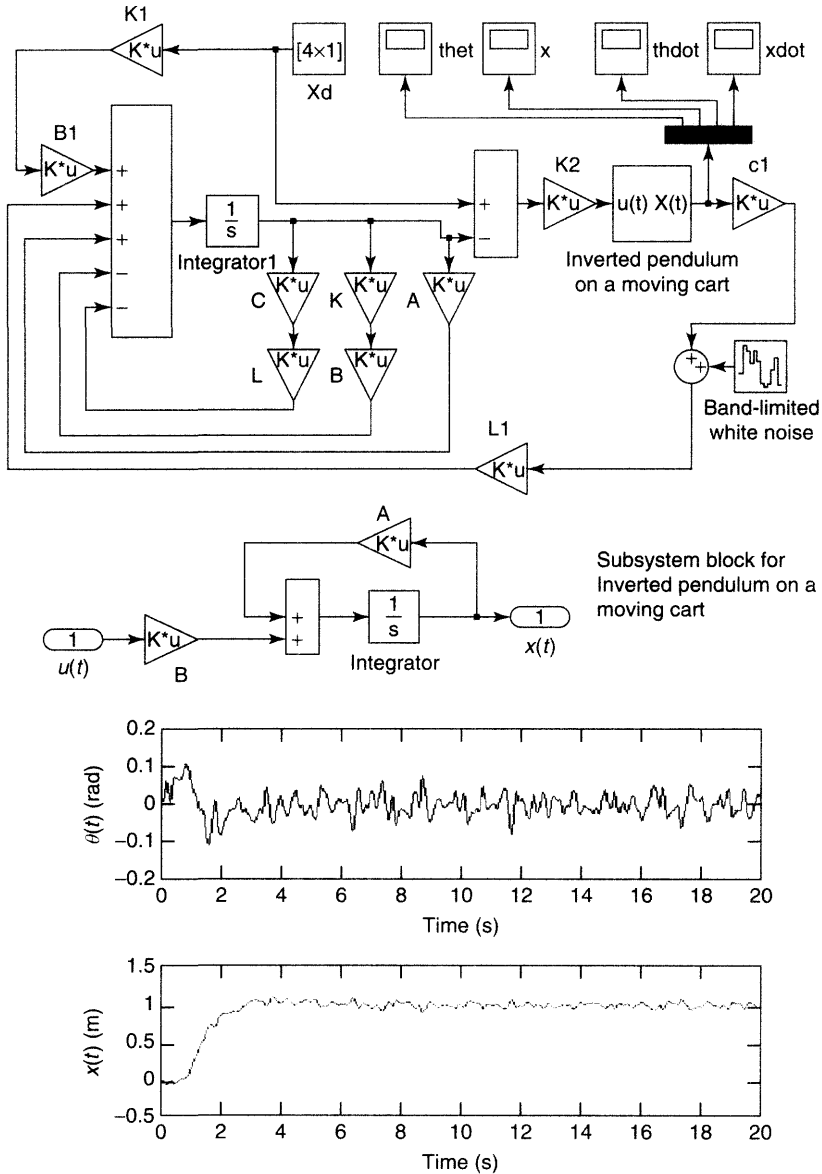


Figure 5.22 Simulation of the inverted pendulum on a moving cart with a full-order compensator and measurement noise with SIMULINK block-diagram

Exercises

5.1. Check the controllability of the plants with the following state-coefficient matrices:

$$(a) \quad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \quad (5.140)$$

$$(b) \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 25 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} \quad (5.141)$$

5.2. As discussed in Section 5.1, an unstable plant is said to be *stabilizable* if all the *uncontrollable* sub-systems have *stable* eigenvalues. Check whether the plants given in Exercise 5.1 are stabilizable.

5.3. If a plant is *stabilizable* (see Exercise 5.2), we can safely ignore the *uncontrollable* sub-systems by removing the rows and columns corresponding to the uncontrollable states from the state coefficient matrices, $\mathbf{A}$ and $\mathbf{B}$. The resulting state-space representation would be controllable, and is called a *minimal realization*. Find the minimal realization of the state coefficient matrices, $\mathbf{A}$ and $\mathbf{B}$ for the plants in Exercise 5.1.

5.4. A *distillation column* in a chemical plant has the following state-coefficient matrices:

$$\mathbf{A} = \begin{bmatrix} -21 & 0 & 0 & 0 \\ 0.1 & -5 & 0 & 0 \\ 0 & -1.5 & 0 & 0 \\ 0 & -4 & 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 6000 & 0 \\ 0 & 0 \\ 0 & 2.3 \\ 0 & 0.1 \end{bmatrix} \quad (5.142)$$

(a) Is the plant controllable?

(b) Suppose we would like to control the plant using *only one* input at a time. Is the plant controllable with only the *first* input, i.e. with $\mathbf{B} = [6000; 0; 0; 0]^T$? Is the plant controllable with only the *second* input, i.e. with $\mathbf{B} = [0; 0; 2.3; 0.1]^T$?

5.5. For the aircraft with lateral dynamics given in Eq. (4.97) in Exercise 4.3:

(a) is the aircraft controllable using both the inputs?

(b) is the aircraft controllable using *only* the aileron input, $\delta_A(t)$?

(c) is the aircraft controllable using *only* the rudder input, $\delta_R(t)$?

5.6. Consider the longitudinal dynamics of a flexible bomber airplane of Example 4.7, with the state-space representation given by Eq. (4.71).

(a) Is the aircraft controllable using both the inputs, $u_1(t)$ and $u_2(t)$?

(b) Is the aircraft controllable using *only* the desired elevator deflection, $u_1(t)$?

(c) Is the aircraft controllable using *only* the desired canard deflection, $u_2(t)$?

5.7. For the aircraft in Exercise 5.5, can you design a full-state feedback regulator which places the closed-loop poles of the aircraft at $s_{1,2} = -1 \pm i$, $s_3 = -15$, $s_4 = -0.8$ using *only one of the inputs*? If so, which one, and what is the appropriate gain matrix?

5.8. For the aircraft in Exercise 5.6, design a full-state feedback regulator using both the inputs and the MATLAB (CST) command *place*, such that the closed-loop poles are located

at $s_{1,2} = -3 \pm 3i$, $s_{3,4} = -1 \pm 2i$, $s_5 = -100$, $s_6 = -75$. Find the maximum overshoots and settling time of the closed-loop initial response if the initial condition vector is $\mathbf{x}(0) = [0; 0.5; 0; 0; 0; 0]^T$.

- 5.9. For the distillation column of Exercise 5.4, design a full-state feedback regulator to place the closed-loop poles at $s_{1,2} = -0.5 \pm 0.5i$, $s_3 = -5$, $s_4 = -21$.
- 5.10. Repeat Exercise 5.9 for the closed-loop poles in a Butterworth pattern of radius, $R = 5$. Compare the initial response of the *first state-variable* (i.e. for $\mathbf{C} = [1; 0; 0; 0]$ and $\mathbf{D} = [0; 0]$) of the resulting closed-loop system with that of Exercise 5.9 for initial condition, $\mathbf{x}(0) = [1; 0; 0; 0]^T$. Which of the two (present and that of Exercise 5.9) regulators requires the larger control input magnitudes for this initial condition?
- 5.11. Consider the turbo-generator of Example 3.14, with the state-space representation given by Eq. (3.117).
- Is the plant controllable using both the inputs, $u_1(t)$ and $u_2(t)$?
 - Is the plant controllable using *only* the input, $u_1(t)$?
 - Is the plant controllable using *only* the input, $u_2(t)$?
 - Design a full-state feedback regulator for the plant using *only* the input, $u_1(t)$, such that the closed-loop eigenvalues are at $s_{1,2} = -2.5 \pm 2.5i$, $s_{3,4} = -1 \pm i$, $s_5 = -10$, $s_6 = -15$.
 - Repeat part (d) using *only* the input, $u_2(t)$.
 - Repeat part (d) using both the inputs, $u_1(t)$ and $u_2(t)$, and the MATLAB (CST) command *place* for designing the multi-input regulator.
 - Re-design the regulators in parts (d)–(f), such that the maximum overshoot and settling time for the output, $y_1(t)$, are less than 0.3 units and 6 seconds, respectively, if the initial condition vector is $\mathbf{x}(0) = [0.1; 0; 0; 0; 0; 0]^T$.
 - Re-design the regulators in parts (d)–(f), such that the closed-loop poles are in a Butterworth pattern of radius, $R = 10$, and compare the closed-loop initial responses and input magnitudes with those of part (g).
- 5.12. Check the observability of the plants with the following state coefficient matrices:

$$(a) \mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0.3 & -0.1 & 0.05 \\ 1 & 0 & 0 \end{bmatrix}; \quad \mathbf{C} = [0 \quad 0 \quad -2] \quad (5.143)$$

$$(b) \mathbf{A} = \begin{bmatrix} 0 & 0.1 & -100 & 4 \\ -250 & -7 & 3 & 50 \\ 0 & 0 & -3.3 & 0.06 \\ 2 & 0 & 0 & 0.25 \end{bmatrix}; \quad \mathbf{C} = [0 \quad 0 \quad 1 \quad 0] \quad (5.144)$$

$$(c) \mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} \quad (5.145)$$

- 5.13. An unstable plant is said to be *detectable* if all the *unobservable* sub-systems have *stable* eigenvalues. Check whether the plants given in Exercise 5.12 are detectable.
- 5.14. If a plant is *detectable* (see Exercise 5.13), we can safely ignore the *unobservable* sub-systems by removing the rows and columns corresponding to the unobservable states from the state coefficient matrices, **A** and **B**. The resulting state-space representation would be observable, and is called a *minimal realization*. Find the minimal realization of the state coefficient matrices, **A** and **B** for the plants in Exercise 5.12.
- 5.15. For the distillation column of Exercise 5.4, the matrices **C** and **D** are as follows:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (5.146)$$

- (a) Is the plant observable?
- (b) Is the plant observable if *only* the *first output* was measured, i.e. $\mathbf{C} = [0; 0; 1; 0]$, $\mathbf{D} = 0$?
- (c) Is the plant observable if *only* the *second output* was measured, i.e. $\mathbf{C} = [0; 0; 0; 1]$, $\mathbf{D} = 0$?
- 5.16. For the aircraft with lateral dynamics given in Eq. (4.97) in Exercise 4.3:
- (a) is the aircraft observable with the output vector, $\mathbf{y}(t) = [p(t); r(t)]^T$?
- (b) is the aircraft observable with the output vector, $\mathbf{y}(t) = [p(t); \phi(t)]^T$?
- (c) is the aircraft observable with the bank-angle, $\phi(t)$, being the only measured output?
- (d) is the aircraft observable with the sideslip-angle, $\beta(t)$, being the only measured output?
- (e) design a full-order observer for the aircraft using *only one* of the state-variables as the output, such that the observer poles are placed at $s_{1,2} = -2 \pm 2i$, $s_3 = -16$, $s_4 = -2$.
- (f) design a full-order compensator based on the regulator of Exercise 5.7, and the observer designed in part (e). Calculate and plot the initial response, $\phi(t)$, of the compensated system if the initial condition is $\mathbf{x}(0) = [0.5; 0; 0; 0]^T$.
- 5.17. Consider the longitudinal dynamics of a flexible bomber airplane of Example 4.7.
- (a) Is the aircraft observable using both the outputs, $y_1(t)$ and $y_2(t)$?
- (b) Is the aircraft observable with *only* the normal acceleration, $y_1(t)$, as the measured output?
- (c) Is the aircraft observable with *only* the pitch-rate, $y_2(t)$, as the measured output?
- (d) Design a full-order observer for the aircraft using *only* the normal acceleration output, $y_1(t)$, such that the observer poles are placed at $s_{1,2} = -4 \pm 4i$, $s_{3,4} = -3 \pm 3i$, $s_5 = -100$, $s_6 = -75$.
- (e) Design a full-order compensator for the aircraft with the regulator of Exercise 5.8 and the observer of part (d). Compute the initial response of the compensated system

with the initial condition given by $\mathbf{x}(0) = [0; 0.5; 0; 0; 0; 0]^T$, and compare with that obtained for the regulated system in Exercise 5.8. Also compare the required inputs for the regulated and compensated systems.

- 5.18. For the distillation column of Exercise 5.15, design a two-output, full-order observer using the MATLAB (CST) command *place* such that the observer poles are placed at $s_{1,2} = -2 \pm 2i$, $s_3 = -5$, $s_4 = -21$. With the resulting observer and the regulator designed in Exercise 5.9, design a full-order compensator and find the initial response of the compensated system for the initial condition $\mathbf{x}(0) = [1; 0; 0; 0]^T$. What are the control inputs required to produce the compensated initial response?
- 5.19. Consider the turbo-generator of Example 3.14, with the state-space representation given by Eq. (3.117).
- Is the plant observable with both the inputs, $y_1(t)$ and $y_2(t)$?
 - Is the plant observable with *only* the output, $y_1(t)$?
 - Is the plant observable with *only* the output, $y_2(t)$?
 - Design a full-order observer for the plant using only the output, $y_1(t)$, such that the observer poles are placed at $s_{1,2} = -3.5 \pm 3.5i$, $s_{3,4} = -5 \pm 5i$, $s_5 = -10$, $s_6 = -15$.
 - Repeat part (d) using only the output, $y_2(t)$.
 - Repeat part (d) using both the outputs, $y_1(t)$ and $y_2(t)$, and the MATLAB (CST) command *place* for designing the two-output full-order observer.
 - Re-design the observers in parts (d)–(f), and combine them with the corresponding regulators designed in Exercise 5.11(g) to form compensators, such that the maximum overshoot and settling time for the compensated initial response, $y_1(t)$, are less than 0.3 units and 6 seconds, respectively, if the initial condition vector is $\mathbf{x}(0) = [0.1; 0; 0; 0; 0; 0]^T$. How do the input magnitudes compare with the required inputs of the corresponding regulators in Exercise 5.11(g)?
- 5.20. Design a reduced-order observer for the aircraft of Exercise 5.16 using the bank-angle, $\phi(t)$, as the only output, such that the observer poles are in a Butterworth pattern of radius, $R = 16$, and combine it with the regulator of Exercise 5.7, to form a reduced-order compensator. Compare the initial response, $\phi(t)$, and the required inputs of the reduced-order compensated system to that of the full-order compensator in Exercise 5.16 (f) with the initial condition $\mathbf{x}(0) = [0.5; 0; 0; 0]^T$.
- 5.21. Design a reduced-order observer for the aircraft of Exercise 5.17 with normal acceleration, $y_1(t)$, as the only output, such that the observer poles are in a Butterworth pattern of radius, $R = 100$, and combine it with the regulator of Exercise 5.8, to form a reduced-order compensator. Compare the initial response and the required inputs of the reduced-order compensated system to that of the full-order compensator in Exercise 5.17(e) with the initial condition $\mathbf{x}(0) = [0; 0.5; 0; 0; 0; 0]^T$.