R and **V** matrices that achieves the desired performance is $\mathbf{R} = 10\mathbf{I}$, and $\mathbf{V} = 907\mathbf{I}$, and the closed-loop response is calculated using *tpbvlti* as follows:

```
>>[u,X,t] = tpbvlti(A,B,zeros(6),10*eye(2),907*eye(6),0,0.2,
  [0 0.1 0 0 0 0]'); y = C*X'; <enter>
```

The resulting outputs $y_1(t)$ and $y_2(t)$, and the required inputs, $u_1(t)$ and $u_2(t)$, are plotted in Figure 8.13. Note that both $y_1(t)$ and $y_2(t)$ are minimized in 0.2 seconds, as desired. The maximum input magnitudes do not exceed ± 0.3 rad.(17°). This design has been carried out after experimenting with various values of v, where $\mathbf{V} = v\mathbf{I}$, and settling with the one ($v = 907$) that gives the minimum magnitudes of $y_1(t_f)$ and $y_2(t_f)$, which are -0.009 m/s^2 and -1.4×10^{-4} rad., respectively.

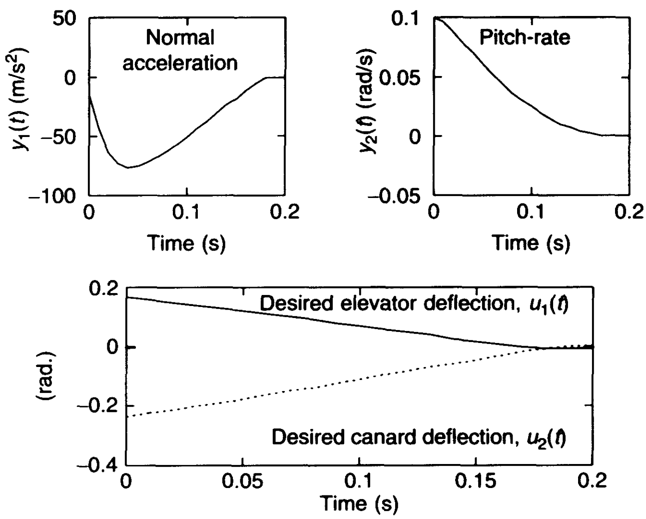

Figure 6.13 Closed-loop response of the flexible bomber aircraft with terminal-time weighted, optimal regulator ($t_f = 0.2$ s)

Exercises

6.1. Design an optimal, full-state feedback regulator for the distillation column whose state-space representation is given in Exercises 5.4 and 5.15, using $\mathbf{Q} = 1 \times 10^{-4}\mathbf{I}$ and $\mathbf{R} = \mathbf{I}$. Determine the initial response of the regulated system to the initial condition, $\mathbf{x}(0) = [1; 0; 0; 0]^T$. Calculate the inputs required for the closed-loop initial response.

6.2. Re-design the optimal regulator for the distillation column in Exercise 6.1, choosing **Q** and **R** such that the settling time of the closed-loop initial response to $\mathbf{x}(0) = [1; 0; 0; 0]^T$

is less than 10 seconds, and the maximum overshoot magnitudes of both the outputs are less than 1×10^{-3} units. Calculate the inputs required for the closed-loop initial response.

6.3. For the aircraft lateral dynamics given in Exercise 4.3, design a two-input optimal regulator with $\mathbf{Q} = \mathbf{I}$ and $\mathbf{R} = \mathbf{I}$. Calculate and plot the initial response, $p(t)$, of the regulated system if the initial condition is $\mathbf{x}(0) = [p(0); r(0); \beta(0); \phi(0)]^T = [0.5; 0; 0; 0]^T$. What are the settling time and the maximum overshoot of the closed-loop initial response? What are the largest input magnitudes required for the closed-loop initial response?

6.4. For the turbo-generator of Example 3.14, with the state-space representation given by Eq. (3.117), design a two-input optimal regulator with $\mathbf{Q} = \mathbf{I}$ and $\mathbf{R} = \mathbf{I}$. Calculate and plot the initial response of the regulated system if the initial condition is $\mathbf{x}(0) = [0.1; 0; 0; 0; 0; 0]^T$. What are the settling time and the maximum overshoots of the closed-loop initial response? What are the largest control input magnitudes required for the closed-loop initial response?

6.5. Re-design the regulator in Exercise 6.4 using output-weighted optimal control such that the maximum overshoot magnitudes of both the outputs in the initial response to $\mathbf{x}(0) = [0.1; 0; 0; 0; 0; 0]^T$ is less than 0.035 units, with a settling time less than 0.6 second, and the required input magnitudes of $u_1(t)$ and $u_2(t)$ should not exceed 0.05 units and 0.4 units, respectively.

6.6. Re-design the regulator for the distillation column to the specifications of Exercise 6.2 using output weighted optimal control.

6.7. Repeat Exercise 6.3 using output weighted optimal control with $\mathbf{Q} = \mathbf{I}$ and $\mathbf{R} = \mathbf{I}$. Compare the initial response and input magnitudes of the new regulator with that designed in Exercise 6.3.

6.8. It is required that the bank-angle, $\phi(t)$, of the aircraft in Exercise 6.3 must track a desired bank-angle given by $\phi_d(t) = 0.1$ rad. Design an optimal tracking system to achieve the desired bank angle in less than 5 seconds with both the input magnitudes less than 0.1 rad., and plot the closed-loop tracking error, $\phi_d(t) - \phi(t)$, if the initial condition of the airplane is zero, i.e. $\mathbf{x}(0) = \mathbf{0}$. Use SIMULINK to investigate the robustness of the tracking system to a random measurement noise in feeding back the roll-rate, $p(t)$.

6.9. Can you design a tracking system for the aircraft in Exercise 6.3 to track a desired constant roll-rate, $p_d(t)$?

6.10. Can you design a tracking system for the turbo-generator of Exercise 6.4 such that a desired state vector is $\mathbf{x}_d(t) = [10; 0; 0; 0; 0; 0]^T$?

6.11. The angular motion of a tank-gun turret [5] is described by the following state-coefficient matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1071 & -46 & -1071 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.7 & -94.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 947 & -17.3 & -947 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7.5 & -101 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 94.3 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 94.3 \end{bmatrix} \tag{6.88}$$

The state-vector is given by $\mathbf{x}(t) = [x_1(t); x_2(t); \ldots; x_8(t)]^T$, where $x_1(t)$ is the *turret azimuth angle*, $x_2(t)$ is the *turret azimuth angular rate*, $x_3(t)$ is the *turret azimuth angular acceleration*, $x_4(t)$ is the *azimuth control hydraulic servo-valve displacement*, $x_5(t)$ is the *turret elevation angle*, $x_6(t)$ is the *turret elevation angular rate*, $x_7(t)$ is the *turret elevation angular acceleration*, and $x_8(t)$ is the *elevation control hydraulic servo-valve displacement*. The input vector, $\mathbf{u}(t) = [u_1(t); u_2(t)]^T$, consists of the input to the *azimuth control servo-valve*, $u_1(t)$, and the input to the *elevation control servo-valve*, $u_2(t)$. Design an optimal tracking system for achieving a constant desired state-vector, $\mathbf{x_d}^c = [1.57; 0; 0; 0; 0.5; 0; 0; 0]^T$, in less than seven seconds, if the initial condition is zero, i.e. $\mathbf{x}(0) = \mathbf{0}$, with the control inputs not exceeding five units.

Plot the initial response and the required control inputs. Using SIMULINK, study the robustness of the tracking system with respect to the following:

(a) measurement noise in the turret azimuth angular acceleration, $x_3(t)$, channel.

(b) measurement noise in the turret elevation angular acceleration, $x_7(t)$.

(c) saturation limits on the two servo-valves, $u_1(t)$ and $u_2(t)$.

6.12. Design a terminal-time weighted optimal regulator for the tank-gun turret if it is desired to move the turret to a zero final state in exactly 0.2 seconds, beginning with an initial condition of $\mathbf{x}(0) = [0.05; 0; 0; 0; 0.01; 0; 0; 0]^T$. Plot the azimuth and elevation angles of the turret, $x_1(t)$, and $x_2(t)$, respectively, and the required control inputs. What are the maximum control input magnitudes?

6.13. For the turbo-generator of Exercise 6.4, design a terminal-time weighted optimal regulator such that the system is brought to a zero final state in exactly 1 second, beginning with the initial condition $\mathbf{x}(0) = [10; 0; 0; 0; 0; 0]^T$. Plot the outputs and the required control inputs.