Exercises

7.1 Repeat Example 7.1 using the MATLAB's random number generator, *rand*, which is a random process with *uniform probability distribution* [1]. What significant differences, if any, do you observe from the normally distributed random process illustrated in Example 7.1?

7.2. The following deterministic signal, $\sin(2t)$, corrupted by a *uniformly distributed* random noise is input to a low-pass filter whose transfer function is given by Eq. (7.15):

$$u(t) = \sin(2t) + 0.5^{*}rand(t) \tag{7.88}$$

where *rand* is the MATLAB's uniformly distributed random number generator (see Exercise 7.1). Determine the filter's cut-off frequency, ω_o, such that the noise is attenuated without too much distortion of the output, $y(t)$, compared to the noise-free signal, $\sin(2t)$. Generate the input signal, $u(t)$, from $t = 0$ to $t = 10$ seconds, using a time step size of 0.01 second, and simulate the filtered output signal, $y(t)$. Draw a Bode plot of the filter.

7.3. Simulate the output to the following noisy signal input to the elliptic filter described in Example 7.3:

$$u(t) = \sin(10t) + \sin(100t) + 0.5^{*}rand(t) \tag{7.89}$$

How does the filtered signal compare with the desired deterministic signal, $\sin(10t) + \sin(100t)$? What possible changes are required in the filter's Bode plot such that the signal distortion and noise attenuation are both improved?

7.4. For a linear system with a state-space representation given by Eqs. (7.40) and (7.41), and the process white noise, $\mathbf{v}(t)$, as the only input (i.e. $\mathbf{u}(t) = \mathbf{z}(t) = \mathbf{0}$), derive the differential equation to be satisfied by the *covariance matrix of the output*, $\mathbf{R_y}(t, t)$. (Hint: use steps similar to those employed in deriving Eq. (7.64) for the covariance matrix of the estimation error $\mathbf{R_e}(t, t)$.)

7.5. An interesting *non-stationary* random process is the *Wiener process*, $\mathbf{w}_i(t)$, defined as the *time integral* of the white noise, $\mathbf{w}(t)$, such that

$$\mathbf{w}_i^{(1)}(t) = \mathbf{w}(t) \tag{7.90}$$

Consider the Wiener process as the output, $\mathbf{y}(t) = \mathbf{w}_i(t)$, of a linear system, into which white noise, $\mathbf{w}(t)$, is input. Using the covariance equation derived in Exercise 7.4, show that the covariance matrix of the Wiener process, $\mathbf{R_{wi}}(t)$, grows linearly with time.

7.6. Design a Kalman filter for the distillation column whose state-space representation is given in Exercises 5.4 and 5.15, using $\mathbf{F} = \mathbf{B}$, $\mathbf{Z} = \mathbf{CC^T}$, $\Psi = \mathbf{0}$, and $\mathbf{V} = 0.01\mathbf{B^TB}$. Where are the poles of the Kalman filter in the s-plane?