Shue, Swan and Rokhsaz [22] introduced the above approach of selecting a positive definite Lyapunov function, and applied it to the wing-rock suppression in a second order system consisting of pure rolling, while Tewari [23] extended the approach to the fifth order system considered in Example 9.6, with additional dynamics of an actuator and the yawing motion – a more realistic model than that of Shue, Swan and Rokhsaz [22]. Assuming a structure for the Lyapunov function that is same as that of the cost function in the performance index [22,23] makes the task of selecting the Lyapunov function easier. The need for nonlinear optimal feedback controller is highlighted in both references [22,23], when it is found that wing-rock suppression by linear feedback control is restricted to only small initial conditions.

Robustness properties of the nonlinear controller with respect to uncertain parameters is an important issue when it is recognized that the nonlinear wing rock aerodynamic model may have significant errors. Such errors generally tend to get amplified by a feedback controller, resulting in a lack of performance and/or stability when implemented in actual conditions. While Tewari [23] ensured robustness with respect to parametric uncertainty by iteratively selecting the controller cost parameters, such that a small variation in the values of aerodynamic and actuator parameters does not lead to a large deviation from nominal performance, more formal methods of guaranteeing robustness in nonlinear optimal control are also available, such as the nonlinear H_∞-optimal control derivation of Wise and Sedwick [24] and van der Schaft [25].

Exercises

9.1. Write a MATLAB M-file for calculating the H_∞-norm of a transfer matrix, $\mathbf{G}(s)$, using the MATLAB command *sigma*. Use the M-file to compute the H_∞-norms of the following transfer matrices:

(a) $\mathbf{G}(s) = [(s+1)/(s^2+2s+3); \quad 1/(s^3+7)]$.

(b) $\mathbf{G}(s) = \begin{bmatrix} 1/(s+2) & 0 \\ -1/(s^2+3s-1) & (s+4)/(s+7) \end{bmatrix}$

(c) $\mathbf{G}(s) = \begin{bmatrix} 10(s+1)/(s+10) & 0 & 0 \\ 0 & (10s+1)/[s(s+1)] & 0 \\ 0 & 0 & s/(s+1) \end{bmatrix}$

9.2. Write a MATLAB M-file for estimating the structured singular value of a transfer matrix, $\mathbf{G}(s)$, using the upper bound of Eq. (9.23), when the uncertainty matrix, $\mathbf{\Delta_A}(s)$, has a block-diagonal structure with distinct blocks [4]. Use the M-file to calculate $\mu(\mathbf{G}(i\omega))$ for the following *constant* transfer matrix:

$$\mathbf{G}(i\omega) = \begin{bmatrix} 4-2i & -(1+i)/2 & -10 \\ -24+6i & 3i & 60-80i \\ -6/5 & -(1+i)/5 & 2(1+i) \end{bmatrix}$$

for each of the following block-diagonal structures for a *constant* uncertainty matrix, $\mathbf{\Delta_A}(s)$:

(a) $\Delta_{\mathbf{A}}(s) = \begin{bmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix}$

(b) $\Delta_{\mathbf{A}}(s) = \begin{bmatrix} \Delta_1 & 0 & 0 \\ 0 & 0 & \Delta_2 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\Delta_{\mathbf{A}}(s) = \begin{bmatrix} \Delta_1 & 0 & 0 \\ \Delta_2 & 0 & 0 \\ 0 & 0 & \Delta_3 \end{bmatrix}$

(d) $\Delta_{\mathbf{A}}(s) = \begin{bmatrix} \Delta_1 & 0 & 0 \\ \Delta_2 & \Delta_3 & 0 \\ 0 & 0 & \Delta_4 \end{bmatrix}$

where Δ_1, Δ_2, Δ_3, Δ_4 are scalar constants. Compare your results with those given in Doyle [4].

9.3. Using the M-file developed in Exercise 9.2, calculate and plot the structured singular value, $\mu(\mathbf{P}_{22}(i\omega))$, as a function of frequency, ω, for a closed-loop system with the following values of the nominal plant transfer matrix, $\mathbf{G_o}(s)$, compensator transfer matrix, $\mathbf{H}(s)$, and the additive uncertainty matrix, $\Delta_{\mathbf{A}}(s)$:

$$\mathbf{G_o}(s) = \begin{bmatrix} 9/(s+1) & -10/(s+1) \\ -8/(s+2) & 9/(s+2) \end{bmatrix}; \quad \mathbf{H}(s) = 1/(0.0159s) \begin{bmatrix} 9(s+1) & 10(s+2) \\ 8(s+1) & 9(s+2) \end{bmatrix}$$

$$\Delta_{\mathbf{A}}(s) = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}$$

where Δ_1, Δ_2 are scalar constants. What is the *maximum value* of $\mu(\mathbf{P}_{22}(i\omega))$ and what is the value of frequency at which it occurs?

9.4. For the rotating spacecraft of Example 6.2, compute a pre-shaped input sequence assuming the only input to be the hub torque, $u_1(t)$, (i.e. $u_2(t) = u_3(t) = 0$), and retaining only the first two flexible modes in Eq. (9.40), with $t_f = 0.1$ second. Plot the hub-rotation angular displacement and velocity due to the pre-shaped input.

9.5. Repeat Exercise 9.4 with $t_f = 1$ second.

9.6. Repeat Exercises 9.4 and 9.5 with the first six flexible modes retained in the input sequence.

9.7. Devise a nonlinear optimal control-law for stabilizing the inverted pendulum on a moving cart with the nonlinear plant's state-equations given by Eqs. (3.17) and (3.18), when the angular motion of the pendulum is large. Calculate and plot the closed-loop initial response if the initial condition is $x(0) = 0$, $x^{(1)}(0) = 0$, $\theta(0) = 1.0$ rad, $\theta^{(1)}(0) = 0.1$ rad/s.